Analysing Guitar Pickup Magnetic Fields

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Introduction

If the last section didn't blow your mind, then this section will! This section pulls together much of what we have discussed and rolls this into some Finite Element Moment (FEM) Analysis so the magnetic fields can be "seen" to a very large degree - even if only in 2D.

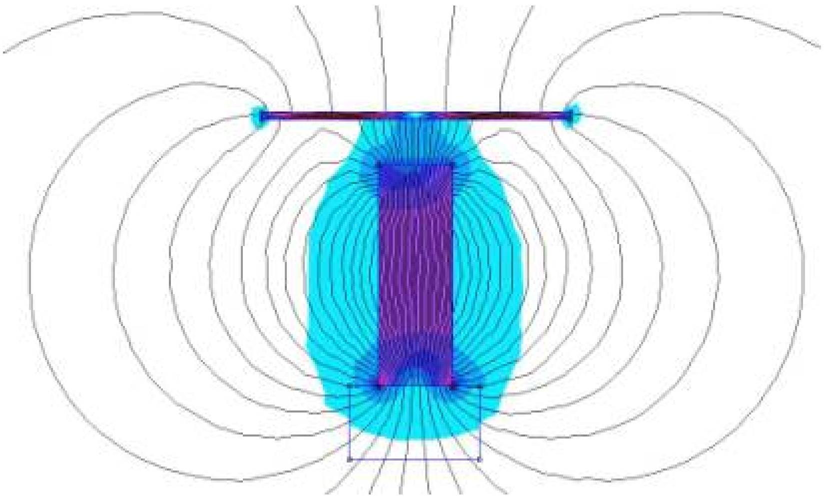
Having created a direct mathematical relationship between a couple of very easy to measure electronic measurements on an electromagnetic pickup and a desired timbre, this is a major breakthrough and it spells out what associated electronics is required for a guitar to get a particular spectral range – in other words the one pickup can be used for Cool Jazz, Cool Blues, Rhythm 'n Blues, Country & Western, and so on – just be changing he load resistor and capacitor values!

While this may be possible in most cases there are other mitigating factors that need to be considered, like background noise, and why it happens, how to get rid of it and why some picks just never seem to work!

Magnetic Fields Part 1

The typical electromagnetic guitar pickup consists of a permanent magnet with a coil of wire wrapped around this magnet, and the (magnetic) strings in near proximity.

When the (magnetic) string vibrates, it alters the magnetic circuit (specifically the reluctance – magnetic resistance) of the magnet and coil combination and because the permanent magnet's magnetic potential field is changed, this induces a current flow in the coil of wire.



The Finite Element Moments (FEM) picture above depicts a typical bar magnet – shown vertically as though you were looking through the end of the pickup - and a part of a string shown above it – shown horizontally.

This is the typical structure for the Strat style (single coil) pickups that have been measured in several different ways in earlier sections of this Website. Any relative vertical movement of the string will alter the reluctance of the magnetic field. As the sting moves away from the magnet, this will cause a minute increase in magnetic reluctance, which will minutely decrease both the magnetic field potential (P), and the self inductance (L) of the coil/magnet assembly (and visa-versa).

A Few Mathematical Transforms

To get a grasp of this mathematically; where P is the Magnetic Potential (or Field Strength), and L is the total Self inductance, this calls for a little multivariable calculus to demonstrate the relationship of the voltage generated in relation to the string movement.

$V(t) = P\, dL/dt + L\, dP/dt$

This is a very general equation, and it does not even have linear or angular dimensions included into it - but it definitely ties output voltage to changes in Magnetic Potential and changes in the total Inductance. Now the fun begins because we can now analyse the instantaneous voltage produced by the relative motion between the pickup and the vibrating string.

As the string cyclically oscillates (normal or vertical to the plane of the pickup magnets), it cyclically changes the magnetic resistance (reluctance) of the pickup magnets' magnetic field, and cyclically changes the inductance of the pickup; and we now know that the maximum voltage (either positive or negative) is when the string is traversing the zero (or quiescent / stable) condition.

Remember when playing on a swing, this is also near perfect sinusoidal motion, and the fastest point is when you go through the swing's resting position, but you come to a temporary halt at a forward limit and a backward limit - just like the guitar string!

So now we know why the maximum voltage is produced when the string is traversing the stable point.

While doing this small signal analysis, we have several other conditions that are currently in our favour because these conditions can be considered as time invariant and therefore we can neglect them and the others can be considered as linear this makes small-signal analysis fairly easy.

Just as a matter of gymnastics - look at the magnetic field above and realise that the string could have been placed below the pickup and the results would be identical. Alternatively, the pickup could have been put in upside-down and it would work equally well. It may not look that pretty - but it will work equally well (simply because the magnetic field is symmetrical about the centre of this magnetic structure)!

The first step that we need to do is to transform our time related maths into frequency related maths, then we can move from a single frequency to a linear spectrum, and then from a linear spectrum to a logarithmic spectrum (because that's the way our ears work)!

Changing from the time domain to the frequency domain is a little tricky - because in mathematics the frequency domain is in radians, and radians are rather poorly understood little creatures.

Basically if you take a circle and measure the diameter, then wrap the diameter around the circumference of the circle; then you will need about 3.14 something diameters to equal the circumference. This constant relationship between circumference and diameter of any circle is called Pi.

We need this relationship so that we can relate a linear dimension (the diameter) with a frequency (rotational) dimension (the circumference). As the radius is the distance from the centre of a circle to the circumference, this is a very nice dimension to use (and it also happens to be half the distance of the diameter)!

This circumference / radius relationship becomes 2* Pi, or about 6.28, and now we are getting somewhere although it may not be obvious - just yet - but look!

We now have a circulatory relation to a linear function - but it is in radians, and this is frustrating, and we need to make it practical, ***and this is the missing link. If we roll the wheel (circle) a full circumference, we have traversed 2 * Pi radians (radius equivalents) to do a full cycle, so there are 2 * Pi radians per cycle.***

Now, if you subtend a right-angled triangle from a point an a circumference to a horizontal line passing through the centre and include the radius (of course), then the angle at the centre from the horizontal will follow the Sine rule as you rotate that point around the circumference, and in reference to the horizontal the point will trace out a sine wave (if the rotations are consistent with time).

So now we have the next link to tie a time related function to a rotational function. In a general form this link can be written as:

Omega = Radians per second

Omega = 2 * Pi * Frequency   Where Frequency is Cycles per second (or Hertz (Hz))

We are now coming out of the forest and into the clearing, because Cycles per second has a more well known unit called the Hertz (after Heindrich Hertz[1])

f(t) = A(max) * Sin(Omega(t) + Ø)   Where Ø is an angular difference

f(t) = A(max) * Sin(2 * Pi * Frequency + Ø)

or

f(t) = A(max) * Sin(2 * Pi * F + Ø)

Applied maths works in a funny way. We firstly observe the situation and then fit various mathematical models until we get one or more that fit the approximations and then we refine it - and that is what we do here!

We know that basically the wave shape of a stringed instrument is a damped Sine wave, and this is a very good starting point. We also know that the voltage generated is 90 degrees (or Pi/2) from zero degrees so we can use a Cosine function as the first approximation. As

V(t) = P dL/dt + L dP/dt

So

V(t) = P * A(Max) * Cos (2 Pi F) + L * B(Max) * Cos (2 Pi F)

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[1] http://www.corrosion-doctors.org/Biographies/HertzBio.htm

There is a lot of common terms here so this can be grossly simplified to:

V(t) = K * Cos(2PiF)

Where K is a sensitivity constant (that is constant for small signals.)

For all those non-believers hook a guitar to a scope and gently thumb-pluck a string near the centre of its span and watch the near perfect sine wave for a short time. *Case almost closed.*

We know that the idle or steady state for a guitar string is when the string is not moving, and there is no sound being generated.

We also know that the instant the string is plucked it has its nominal maximum exertion form the idle state.

We also know that in nature all life decays exponentially - so it makes sense that after the string is plucked, it decays to its' stable state with a defined time constant that is directly related to the 'sustain' and this has a direct inverse relation to the energy lost in holding the string to the fret.

This is why open strings subtended by the bridge and machine head (nut) have a much longer sustain time than a string subtended by the bridge and a fret (by a finger).

The finger absorbs a small amount of energy that dulls the tone and considerably shortens the sustain time constant. In fairly simple maths, a decaying step is described by:

Output (t) = Max * Exp(1 - t/Tc) Where Tc is a known time constant

In putting these two equations together we have a rather simplified approximation for a damped sine wave from a plucked string:

V(t) = (Max * Exp(1 - t/Tc)) * K * Cos(2PiF)

There is little point in getting any deeper in this area as it will not prove anything substantial - but it does close the case for the first round of small signal analysis!

As the movement of the string is small, then the change in reluctance can be taken as linear with relative string positioning.

Virtually all guitar pickup analysis stops here (usually much earlier), but in reality, we are just starting to get a better appreciation of what is really going on - and now is the time to move on to slightly larger signal analysis.

That "Warm" Sound - from Large-Signal Analysis

If the string movement causes the change in reluctance to be non-linear, then the time-dependent voltage generated will also be non-linear with relative string positioning.

Because the air gaps in these magnetic transducers are relatively big, and the relative string movements of fundamental mode vibrations are sometimes also relatively big, then for most fundamental string vibrations, there is a non-linear reluctance/movement relationship and the wave shape from the transducer in these cases will be non-linear.

Because of the “distance squared” relationship between the magnetic potential field and the relatively moving string, the output will be a slightly tilted sine wave that looks more like a sawtooth than a clean sine wave.

This might be difficult to understand - until you realise that the output is greater while the string is deeper into the magnetic field and lesser as it is lesser into the magnetic field - and remember that the voltage is a direct product of the ***instantaneous relative movement*** of the string in the magnetic potential - ***not the instantaneous position*** of the string in the magnetic field. (But they are related!)

If you have ever wondered why an electric guitar often sounds like a violin then here is the answer: The violin's bow, when drawn across the strings causes the strings to hop and recover on their resonant frequency - producing a pronounced sawtooth waveform. This is very similar to that of an electric guitar - except that a violin has more timbre (is richer in harmonics).

For those that are mathematically inquisitive, the Laplace (pronounced la-plarze) Transform of a sawtooth waveform consists entirely of even harmonics, and that sounds very warm. I am not going to do the maths on this as it is available from a multitude of sources. Like bees around honey - musos generally love warm notes.

There are a few problems with putting the pickup very close to the strings in that the strings can be 'pulled' by the intrinsic magnetic field, and/or they can buzz when the extremities of the movements actually hit the pole faces in the pickups. The 'pulling' of the strings causes non-harmonic notes to be generated that are slightly flat - and most musos hate these sounds!

So an electric guitar can and does sound like a violin but substantially without the abundance of warm even order harmonics that come from a violin being 'caressed' with a bow.

The musos want much more (even harmonics) from an electric guitar and this can be done in the amplifier with relative ease. This is another topic and I will cover this in detail in another area - devoted to amplifiers - and what really happens in there!

Interactive Issues

The problem with the simple pickup is that the magnetic field is very leaky and because of that, not only is the field very susceptible to external interference like florescent lights but the windings have to have an incredibly high number of turns to pick up the fluctuations in field strength, and this results in a relatively high self inductance and self resistance.

Typically these single coil transducers have a self-inductance of about 3 Henrys and an internal resistance of about 6000 ohms – based on a typical 10,000 turn winding, and they produce typically 100 mV rms.

To get a normalised approach to this, the:
Ar is about 6,000 / (10,000 ^2) = 60 u ohms,

Al is about 3/(10,000^2) = 30 nH,

And the time constant is about 0.5 msec or seen another way, about 250 Hz.

As we now know that these transducers appear as a voltage source with a coil in series, then the upper frequency response knee can be set by the load resistance. If 1 Mohm is used as a reference, then the upper knee frequency is R / (2*PI *L) or 159/L kHz. In this case of 3 H it is about 53 kHz.

It is more common to have a 200 k ohm load so the knee is much closer to 10.6 kHz but in practice this will not happen because of excessive capacitance!

To compound the matter, the upper frequency response is limited by self-capacitance – typically about 70 pF – bringing the resonance to about 8 kHz. With some cable capacitance this resonance can come down remarkably. Say 1500 pF in cable leads and the self-resonance is 1.959 kHz and with a 0.022 uF “Tone” capacitor, across the cable the self-resonance is back to about 600 Hz.

Now with a little understanding on why these external; forces cause so many apparently interactive frequency / spectral response issues it is not wonder that so many of these pickups are so ‘twiddle’ fussy – it takes a long time to ‘get the right setting with a guitar - even on stage! This is one reason why some pickups are ‘so ugly’ – and not the looks of them.

Comments and Corrections are welcome